VNCAP
Validated Numerics for Computer-Assisted Proofs

International Workshop on Validated Numerics for Computer-Assisted Proofs

Computer assisted proofs for invariant manifolds and their use in connecting orbit problems

T-points in the Lorenz system

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Joint work with Sheldon Newhouse and Russell Peele (FAU)

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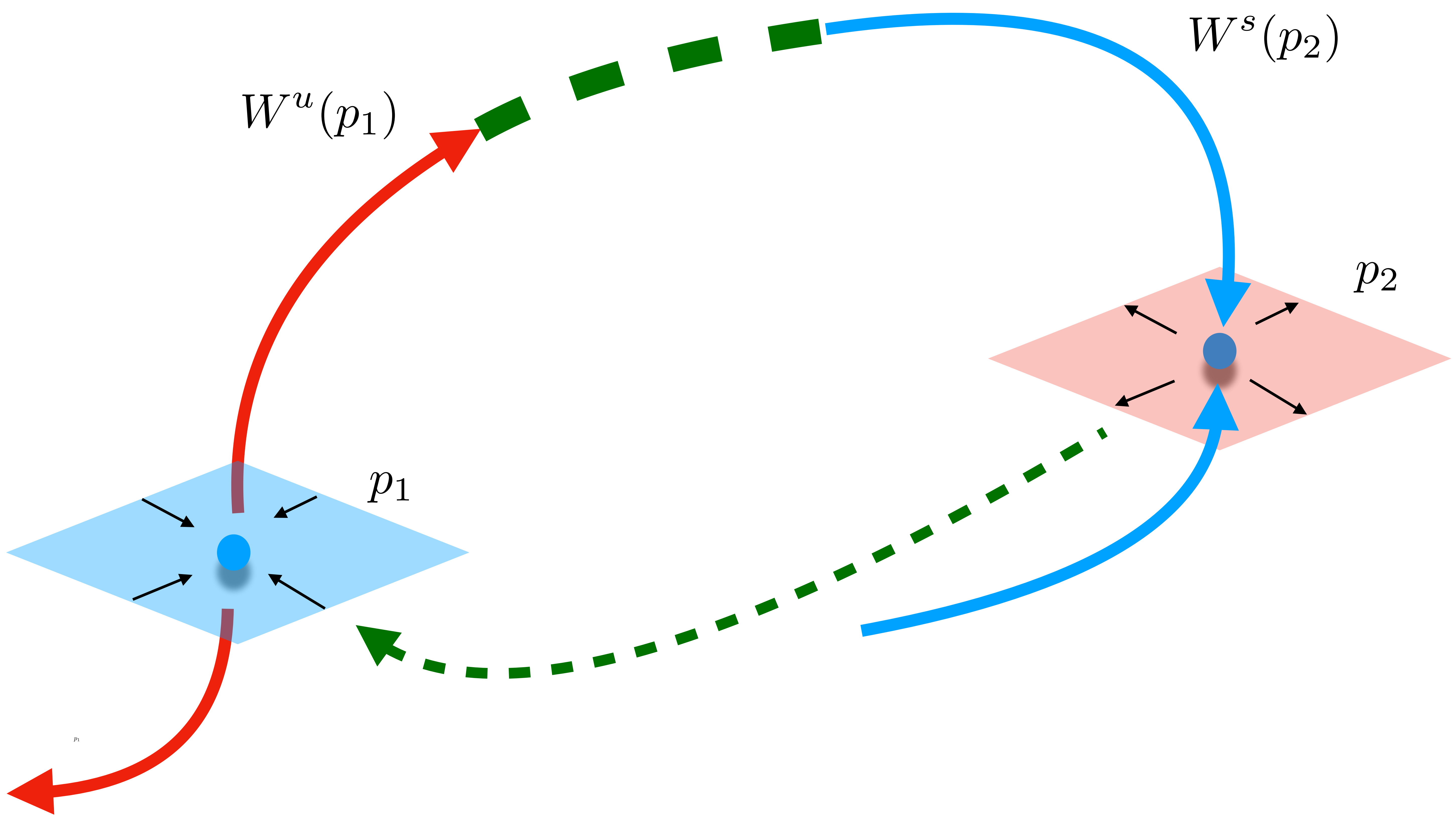
ICMS, Bayes Center
Edinburgh
Scotland

FAU
FLORIDA ATLANTIC UNIVERSITY

ICMS NSF

T-points: $\mathbb{R}^3$



T-points:

Saddle to saddle-focus:

Bykov V V 1978 On the structure of a neighbourhood of a separatrix cycle with a saddle-focus *Methods of Qualitative Theory of Differential Equations* vol 133 (Gorky: Gorky State University) pp 3–32 (in Russian)

Glendinning P and Sparrow C 1986 T-points: a codimension two heteroclinic bifurcation *J. Stat. Phys.* **43** 479–88

Theorem which says that if $(\bar{\sigma}, \bar{\rho})$ is a T-point (saddle to saddle-focus) then there exists a tube about the loop so that for all nearby parameters there is a hyperbolic invariant set in the tube, containing a countably infinite set of periodic orbits.

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Also: there exist one parameter families of co-dimension one (homoclinic) bifurcations which "terminate" at the T-point.

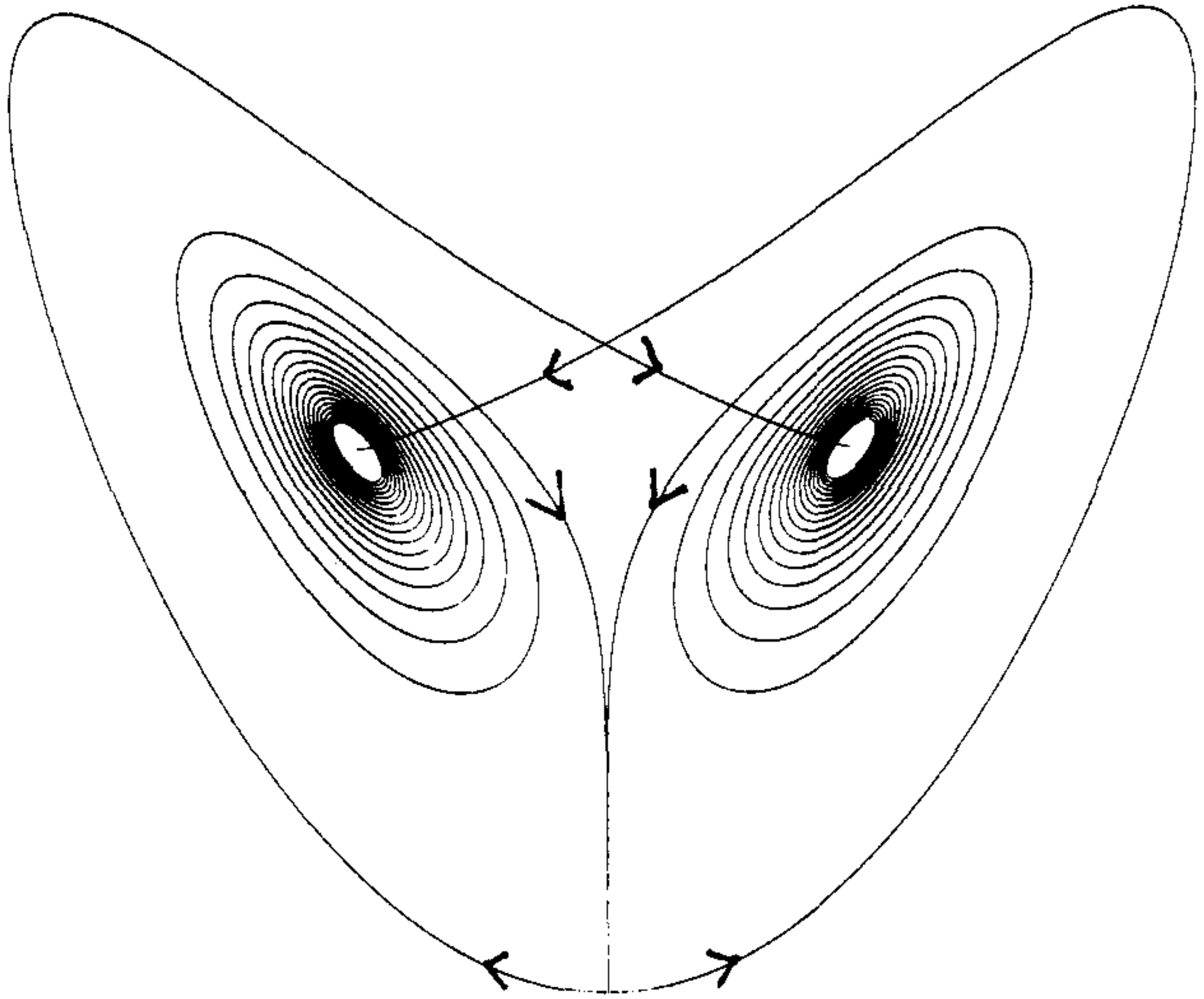
T-points:

Numerical Evidence for T-points in Lorenz:

Petrovskaya N V and Yudovich V I 1980 Homoclinic loops of the Saltzman-Lorenz system *Methods of the Qualitative Theory of Differential Equations* (Gorky: Gorky State University) pp 73–83 (in Russian)

Alfsen K H and Frøyland J 1985 Systematics of the Lorenz model at $\sigma = 10$ *Phys. Scr.* **31** 15–20

$$f(x, y, z, \sigma, \beta, \rho) = \begin{pmatrix} \sigma(y - x) \\ x(\rho - z) - y \\ xy - \beta z \end{pmatrix}$$



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Jennifer L. Creaser, Bernd Krauskopf, and Hinke M. Osinga, *α-flips and T-points in the Lorenz system*, Nonlinearity **28** (2015), no. 3, R39.

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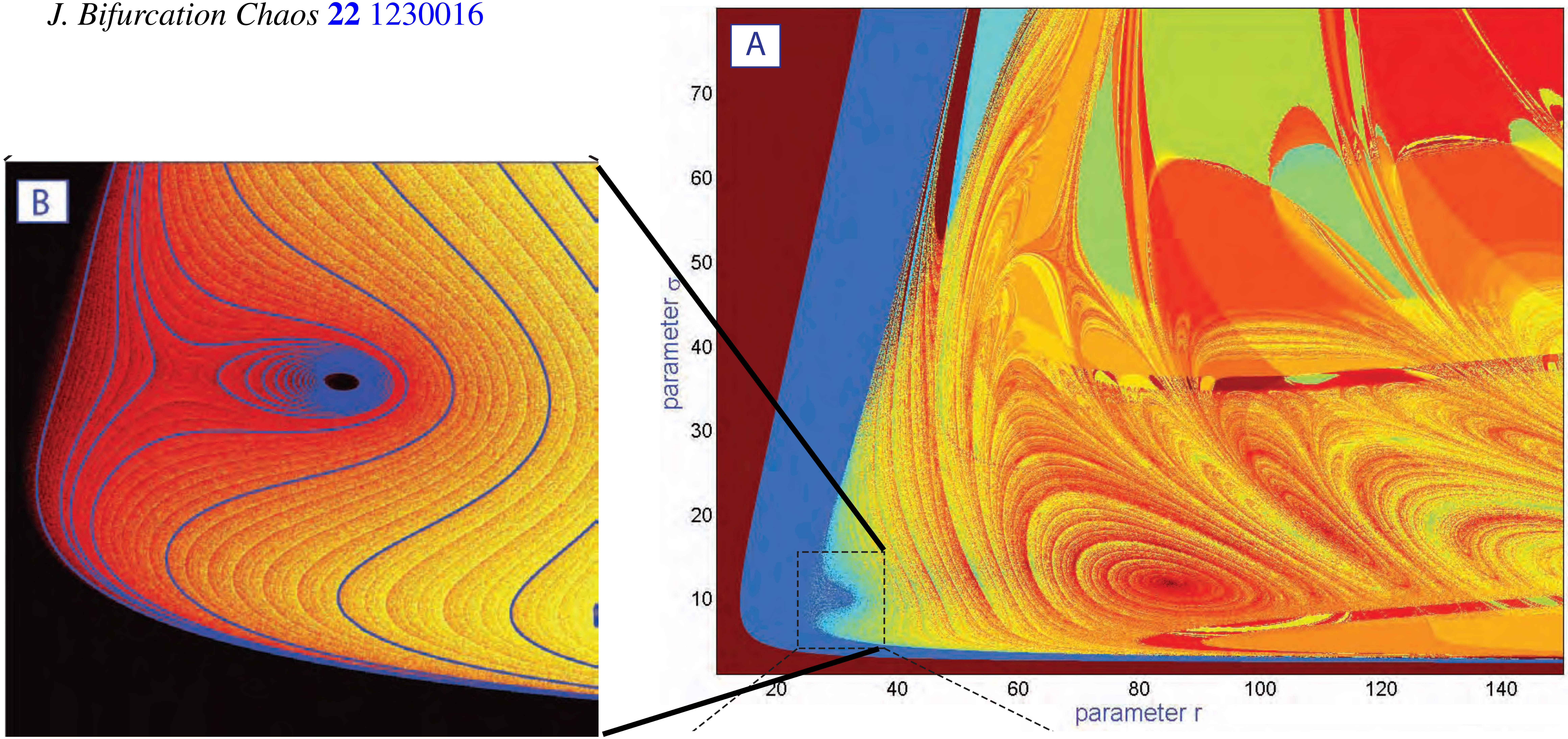

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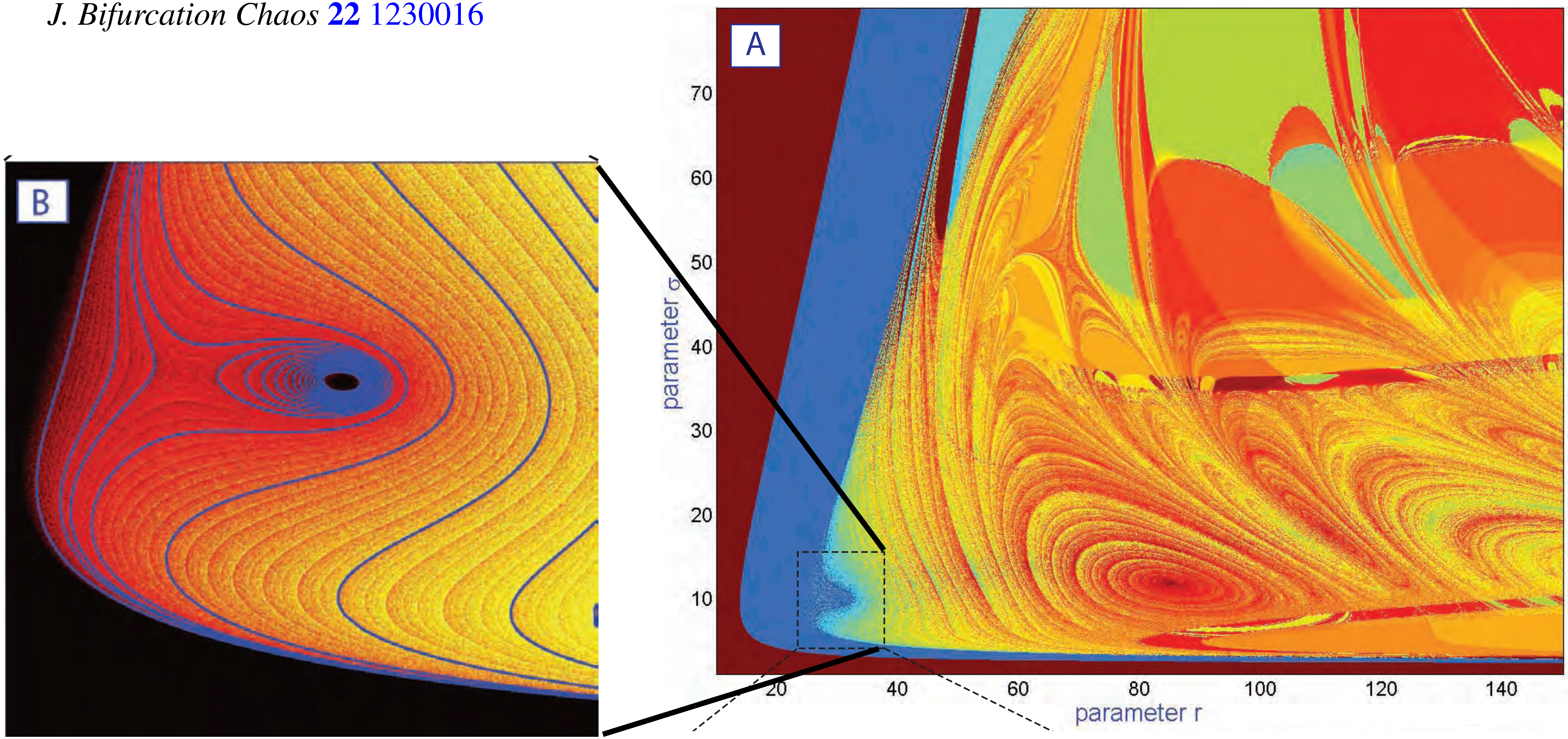

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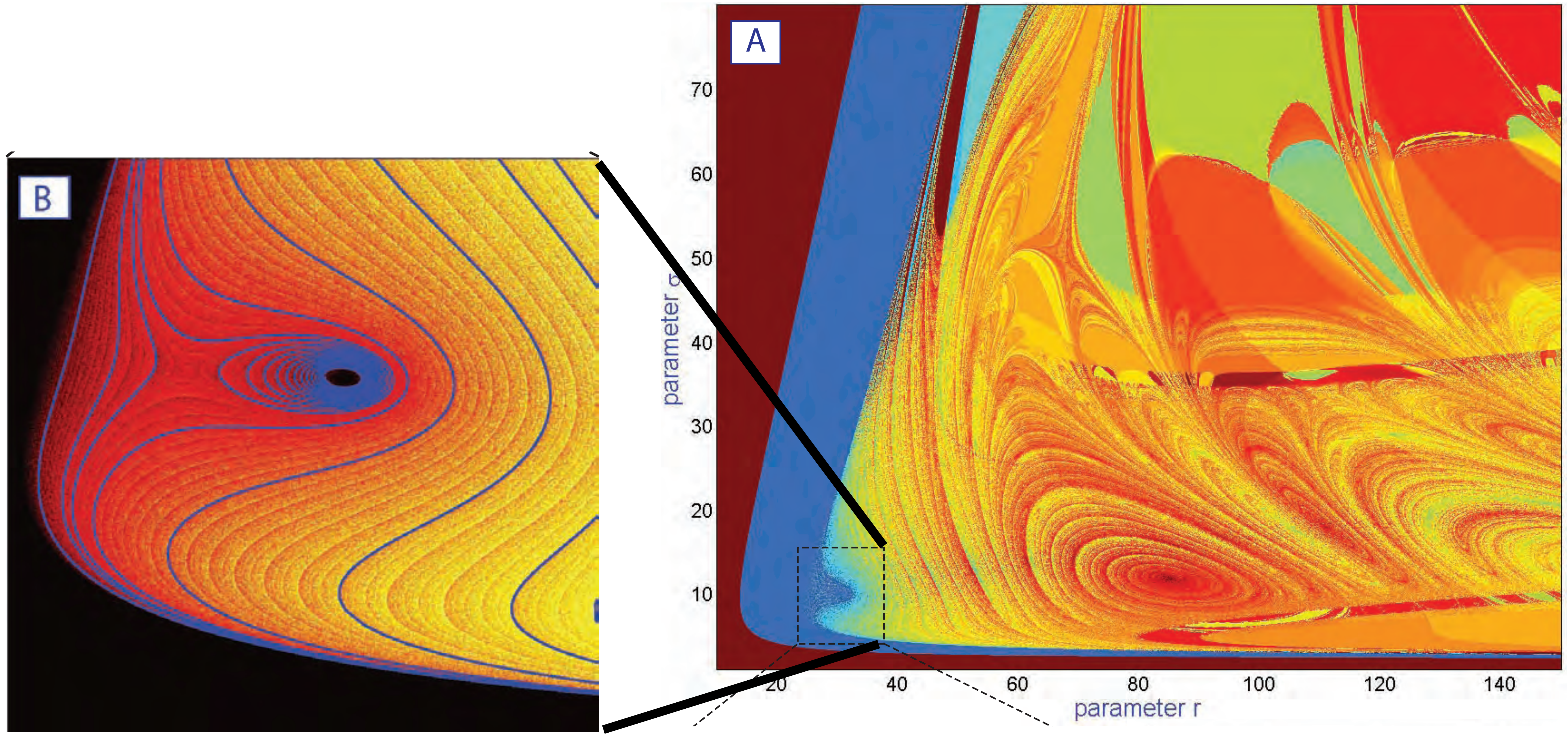

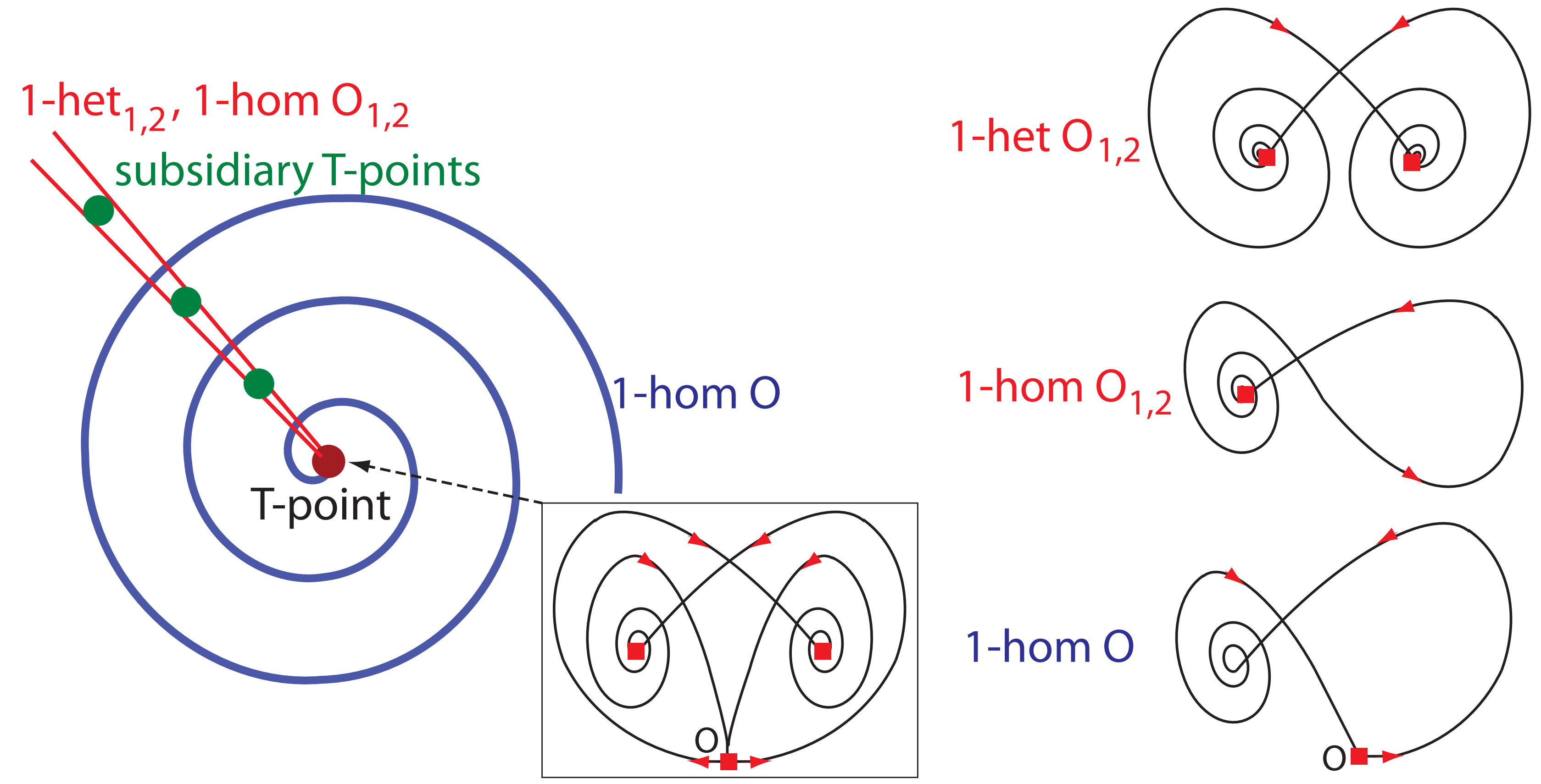

Figure 1. Caricature of the bifurcation unfolding of the ordinary T-point for symmetric Lorenz-like systems with a closed heteroclinic connection evolving both saddle-foci and the saddle. Point out that with each revolution approaching the T-point along the curve (1-hom O), the number of turns of the one-dimensional separatrix of the saddle, O, around the saddle-focus increases by one in the homoclinic loop and becomes infinite at the T-point.

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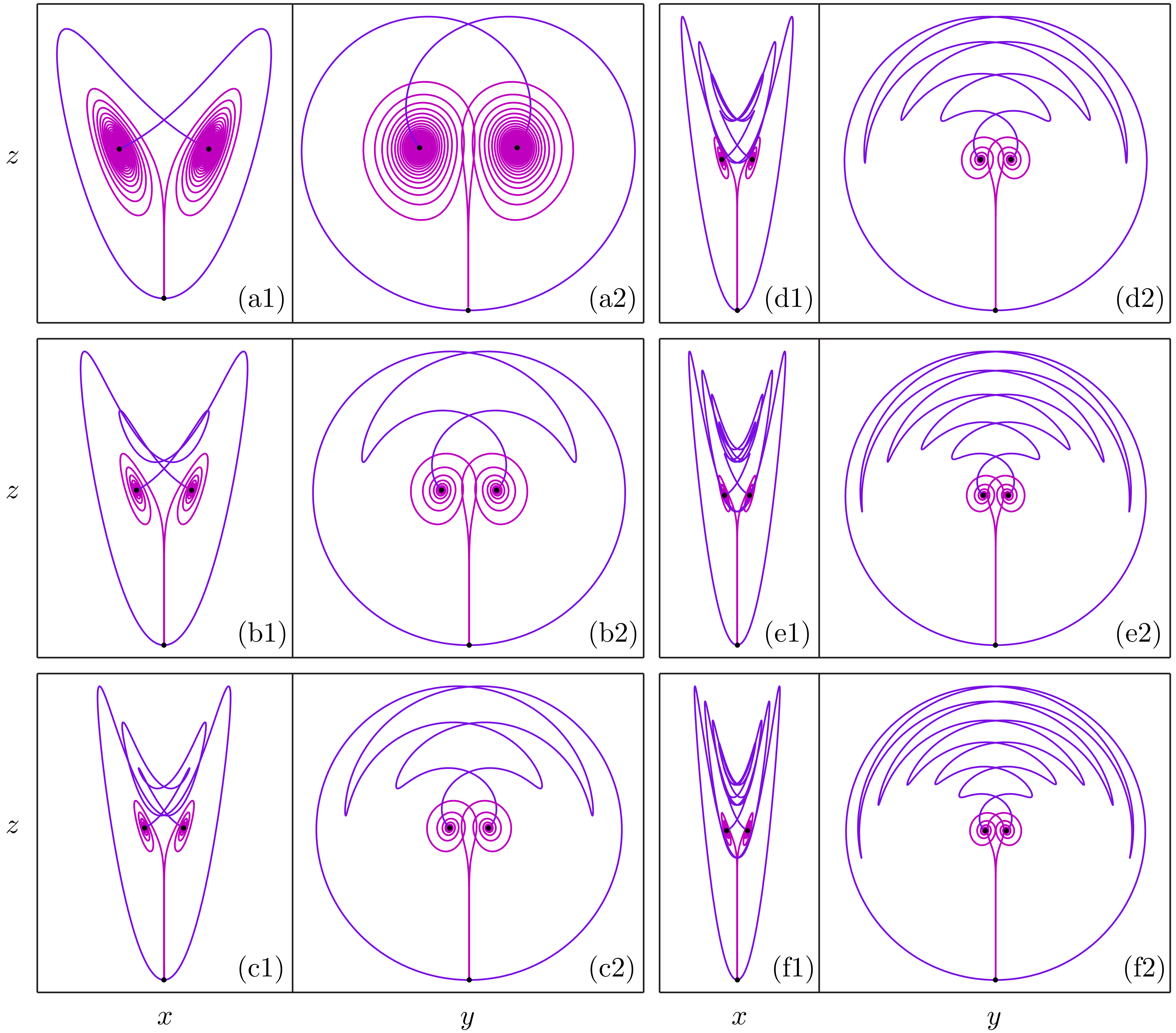

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Table 3. The α-flip values ρ_n^f and $\ln(\rho_n^T)$ for $\sigma = 10$, and the T-point values σ_n^T, ρ_n^T and $\ln(\rho_n^T)$ for $n = 1, \ldots, 25$. The differences $\Delta_n^\sigma = \sigma_n^T - \sigma_{n-1}^T$ and $\Delta_n^{\ln(\rho)} = \ln(\rho_n^T) - \ln(\rho_{n-1}^T)$ are shown, as well as the ratios $\Delta_n^\sigma/\Delta_{n-1}^\sigma$ and $\Delta_n^{\ln(\rho)}/\Delta_{n-1}^{\ln(\rho)}$.

n	ρ_n^f	$\ln(\rho_n^f)$	σ_n^T	Δ_n^σ	$\Delta_n^\sigma/\Delta_{n-1}^\sigma$	ρ_n^T	$\ln(\rho_n^T)$	$\Delta_n^{\ln(\rho)}$	$\Delta_n^{\ln(\rho)}/\Delta_{n-1}^{\ln(\rho)}$
1	29.7191	3.3918	10.1673			30.8680	3.4297		
2	57.6817	4.0549	11.8279	1.6606		85.0292	4.4430	1.0133	
3	89.5040	4.4943	12.9661	1.1382	0.6854	164.1376	5.1007	0.6577	0.6491
4	124.4925	4.8242	13.8424	0.8763	0.7699	267.6019	5.5895	0.4888	0.7432
5	162.2217	5.0890	14.5578	0.7154	0.8164	394.9239	5.9787	0.3892	0.7962
6	202.4018	5.3103	15.1633	0.6055	0.8464	545.6849	6.3020	0.3233	0.8308
7	244.8209	5.5005	15.6888	0.5254	0.8677	719.5287	6.5786	0.2766	0.8553
8	289.3161	5.6675	16.1530	0.4642	0.8835	916.1474	6.8202	0.2416	0.8735
9	335.7575	5.8164	16.5689	0.4159	0.8959	1135.2714	7.0346	0.2144	0.8877
10	384.0383	5.9507	16.9456	0.3767	0.9058	1376.6619	7.2274	0.1928	0.8990
11	434.0686	6.0732	17.2900	0.3443	0.9140	1640.1053	7.4025	0.1751	0.9082
12	485.7715	6.1857	17.6070	0.3171	0.9208	1925.4086	7.5629	0.1604	0.9159
13	539.0803	6.2899	17.9009	0.2938	0.9266	2232.3962	7.7108	0.1479	0.9224
14	593.9360	6.3868	18.1746	0.2737	0.9317	2560.9072	7.8481	0.1373	0.9280
15	650.2864	6.4774	18.4308	0.2562	0.9360	2910.7937	7.9762	0.1281	0.9328
16	708.0843	6.5626	18.6716	0.2408	0.9399	3281.9182	8.0962	0.1200	0.9370
17	767.2874	6.6429	18.8988	0.2272	0.9433	3674.1533	8.2091	0.1129	0.9408
18	827.8569	6.7188	19.1137	0.2150	0.9463	4087.3797	8.3157	0.1066	0.9441
19	889.7575	6.7909	19.3177	0.2040	0.9490	4521.4856	8.4166	0.1009	0.9470
20	952.9566	6.8596	19.5118	0.1941	0.9515	4976.3659	8.5125	0.0959	0.9497
21	1017.4240	6.9250	19.6969	0.1851	0.9537	5451.9213	8.6037	0.0913	0.9521
22	1083.1319	6.9876	19.8739	0.1769	0.9557	5948.0582	8.6908	0.0871	0.9543
23	1150.0540	7.0476	20.0433	0.1694	0.9576	6464.6875	8.7741	0.0833	0.9563
24	1218.1662	7.1051	20.2058	0.1625	0.9593	7001.7246	8.8539	0.0798	0.9581
25	1287.4456	7.1604	20.3620	0.1562	0.9609	7559.0891	8.9305	0.0766	0.9598

T-points:

Analytical Results for T-points in Lorenz:

Lorenz Equations Part II: Randomly Rotated Homoclinic Orbits and Chaotic Trajectories

Xinfu Chen (1995)

Proves that for any positive β and each $N \in \mathbb{N}$, there is a ρ large enough so that the Lorenz system has an orbit homoclinic to the origin which rotates around the z-axis $N/2$ times.

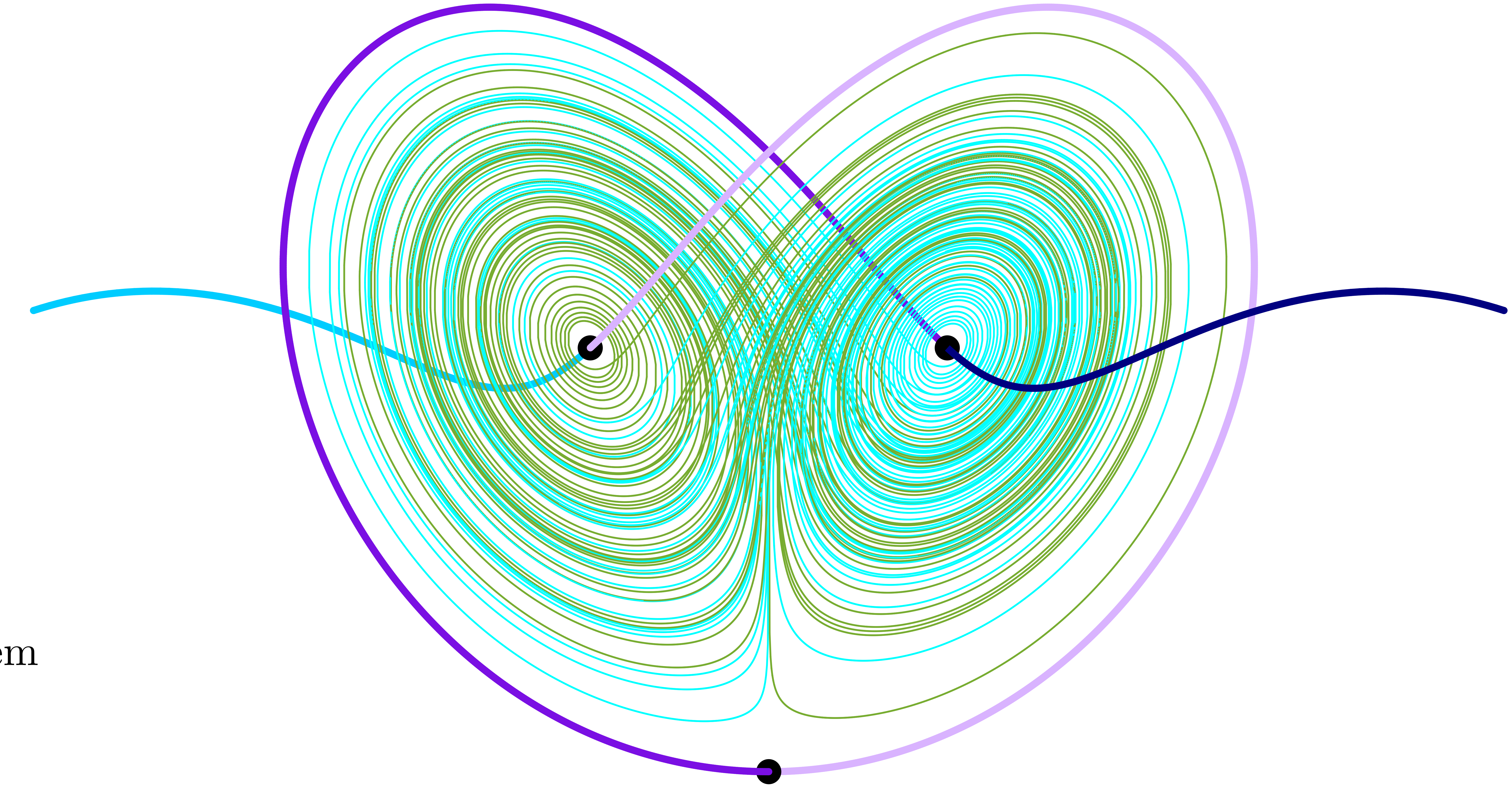
Around Smale's 14th Problem

Tali Pinsky (2022 Archive)

Uses Chen's theorem to show

Theorem 2.4. *There exists a point in the parameter space at which there exists a heteroclinic trefoil for the Lorenz equations.*

T-points:



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T-points:

- T-points of Pinsky/Chen seem to be related to the asymptotics suggested by Osinga/Krauskof/Creaser. These are perturbative in nature - prove them by hand.
- We are interested in proving the existence of the T-points in the non-perturbative regime.

This is an excellent example of how computer assisted and perturbative methods work together.

Main result: Let $\beta = 8/3$. The Lorenz system has a T-pooint $(\bar{\sigma}, \bar{\rho})$ with

$$\bar{\sigma} = 10.167289454815913 \pm 8.5 \times 10^{-12}$$
$$\bar{\rho} = 30.868087468178345 \pm 8.5 \times 10^{-12}$$


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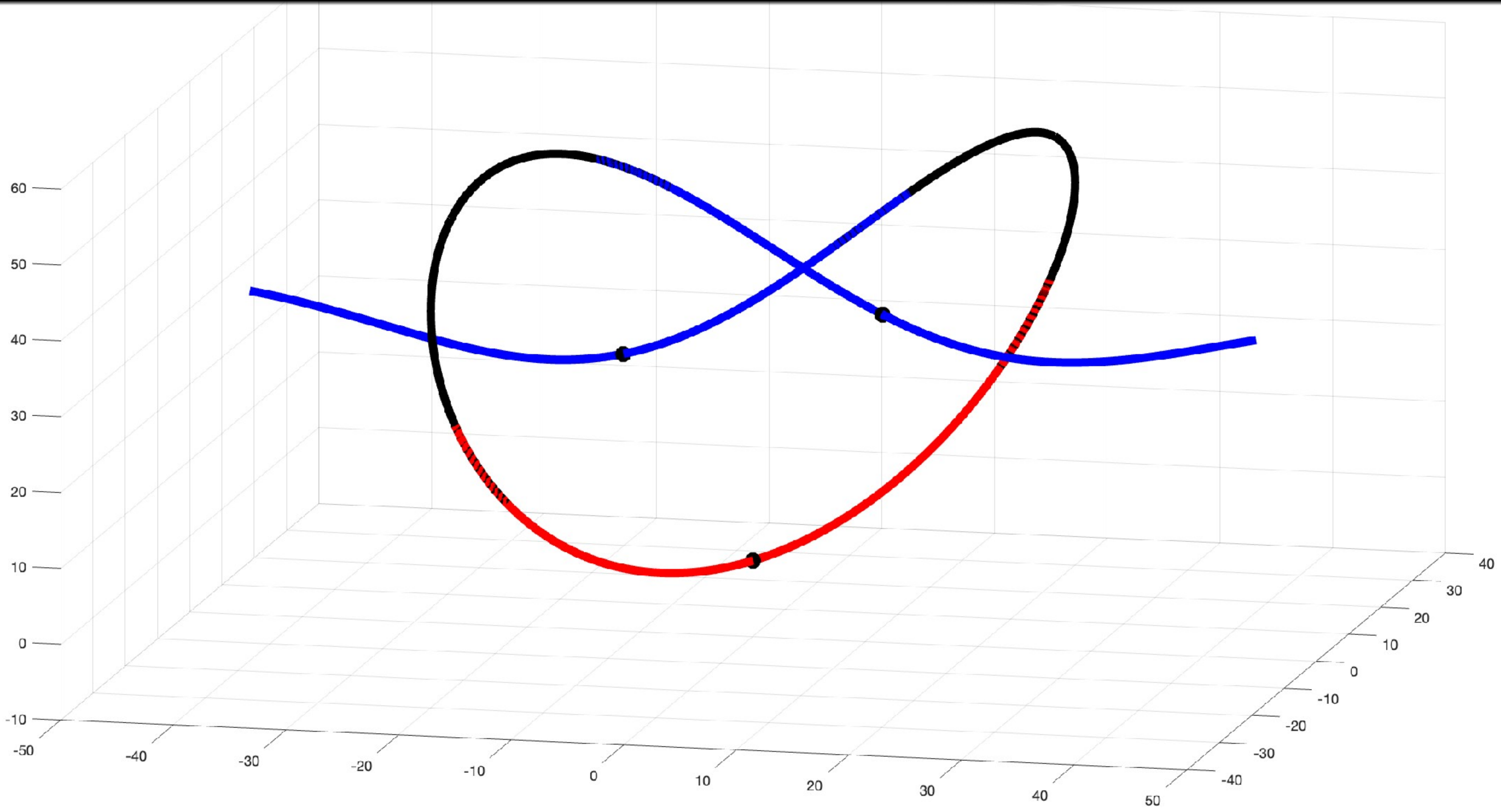
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Set up for the computer assisted proof

Step 1: local invariant manifolds. (Parameterization method for 1d manifolds)

Suppose that $\Omega \subset \mathbb{R}^d$ is an open set and let $f \colon \Omega \to \mathbb{R}^d$ be a smooth vector field. We claim that if $\lambda \neq 0$ and $P \colon [-1, 1] \to \mathbb{R}^d$ is a smooth function satisfying

$$P'(u)\lambda u = f(P(u)), \tag{3}$$

then P parameterizes a one dimensional subset of a stable or unstable manifold, for some equilibrium solutions of f.

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Proposition 2.1 (Parameterization method for a 1d stable/unstable manifold of a vector field)**.** *Suppose that $P\colon [-1,1] \to \mathbb{R}^d$ is a smooth mapping and that $\lambda \in \mathbb{R}$. Assume that $\lambda, \|P'(0)\| \neq 0$, and that P takes values in Ω. If $P(u)$ solves Equation (3) on $(-1,1)$ then*

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- *$p_0 = P(0)$ is an equilibrium solution for the vector field f.*
- *λ is an eigenvalue of $Df(p_0)$ with eigenvector $P'(0)$.*

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- *$p_0 = P(0)$ is an equilibrium solution for the vector field f.*
- *λ is an eigenvalue of $Df(p_0)$ with eigenvector $P'(0)$.*
- *If $\lambda < 0$ then the image of P is a forward invariant submanifold of the stable manifold attached to p_0. If $\lambda > 0$ then the image of P is a backward invariant submanifold of the unstable manifold attached to p_0. In either event the submanifold is tangent at p_0 to the stable/unstable eigenspace spanned by $P'(0)$.*

Main Result

(Parameterization method for 1d manifolds)

$$P'(u)\lambda u = f(P(u)), \qquad u \in (-1, 1) \tag{3}$$

- $p_0 = P(0)$ *is an equilibrium solution for the vector field* f*.*

Evaluating Equation (3) at $u = 0$ leads to

$$P'(0) \cdot \lambda \cdot 0 = f(P(0)).$$

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- $p_0 = P(0)$ *is an equilibrium solution for the vector field* f.

Differentiating Equation (3) with respect to u we obtain

$$P''(u)\lambda u + \lambda P'(u) = Df(P(u))P'(u).$$

Evaluating at $u = 0$ and letting $P'(0) = \xi$ we have that $\lambda\xi = Df(p_0)\xi$. Since $\xi \neq 0$ by hypothesis, we have that (λ, ξ) is an eigenpair for $Df(p_0)$.

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Suppose now that $\lambda < 0$ and choose $u_0 \in [-1, 1]$ with $u_0 \neq 0$.

Define

$$\gamma(t) = P(u_0 e^{\lambda t}).$$

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$$\lim_{t \to \infty} \gamma(t) = P(0)$$

with $P(0)$ the equilibrium solution.

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To see that $\gamma(t)$ solves the differential equation, note that for $t > 0$ we have that

$$\frac{d}{dt}\gamma(t) = P'(u_0 e^{\lambda t}) u_0 \lambda e^{\lambda t}.$$

Let $u_0 e^{\lambda t} = u_1 \in [-1, 1]$

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Suppose now that $\lambda < 0$ and choose $u_0 \in [-1, 1]$ with $u_0 \neq 0$.

$$\begin{aligned} P'(u_0 e^{\lambda t}) u_0 \lambda e^{\lambda t} &= P'(u_1)\lambda u_1 \\ &= f(P(u_1)) \\ &= f(P(e^{\lambda t})) \\ &= f(\gamma(t)). \end{aligned}$$

To see that $\gamma(t)$ solves the differential equation, note that for $t > 0$ we have that

$$\frac{d}{dt}\gamma(t) = P'(u_0 e^{\lambda t}) u_0 \lambda e^{\lambda t}.$$

Let $u_0 e^{\lambda t} = u_1 \in [-1, 1]$

Main Result

Define the map

$$F(\lambda, \mu, T, \sigma, \rho, \mathbf{v}_0, P, Q, R) = \left(\begin{array}{c} P(r) - Q(-\tau) \\ Q(\tau) - R(-r) \\ \|P'(0)\|^2 - C \\ \|R'(0)\|^2 - C \\ \lambda u P'(u) - f_{\sigma,\rho}(P(u)) \\ Q(t) - \mathbf{v}_0 - T\int_0^t f_{\sigma,\rho}(Q(s))\, ds \\ \mu s R'(s) - f_{\sigma,\rho}(R(s)) \end{array} \right)$$

where we fix $r, \tau, C > 0$ and seek a solution with

$$\lambda > 0 \qquad \mu < 0 \qquad T > 0$$

$$\mathbf{v}_0 = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

and

$$P(u) = \begin{pmatrix} P_1(u) \\ P_2(u) \\ P_3(u) \end{pmatrix} \quad Q(t) = \begin{pmatrix} Q_1(t) \\ Q_2(t) \\ Q_3(t) \end{pmatrix}$$

$$R(s) = \begin{pmatrix} R_1(s) \\ R_2(s) \\ R_3(s) \end{pmatrix}$$ analytic on the unit disk in $\mathbb{C}$.

Main Result

Define the map

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Truncate and look for polynomial approximations to the power series of P, Q, R.

The truncated problem is a large system of (quadratic!) polynomial equaitons.

Unknown parameters appear explicitly.

Use a Newton-Kantorovich Theorem to show there is a true solution near the approximate solution.

Main Result

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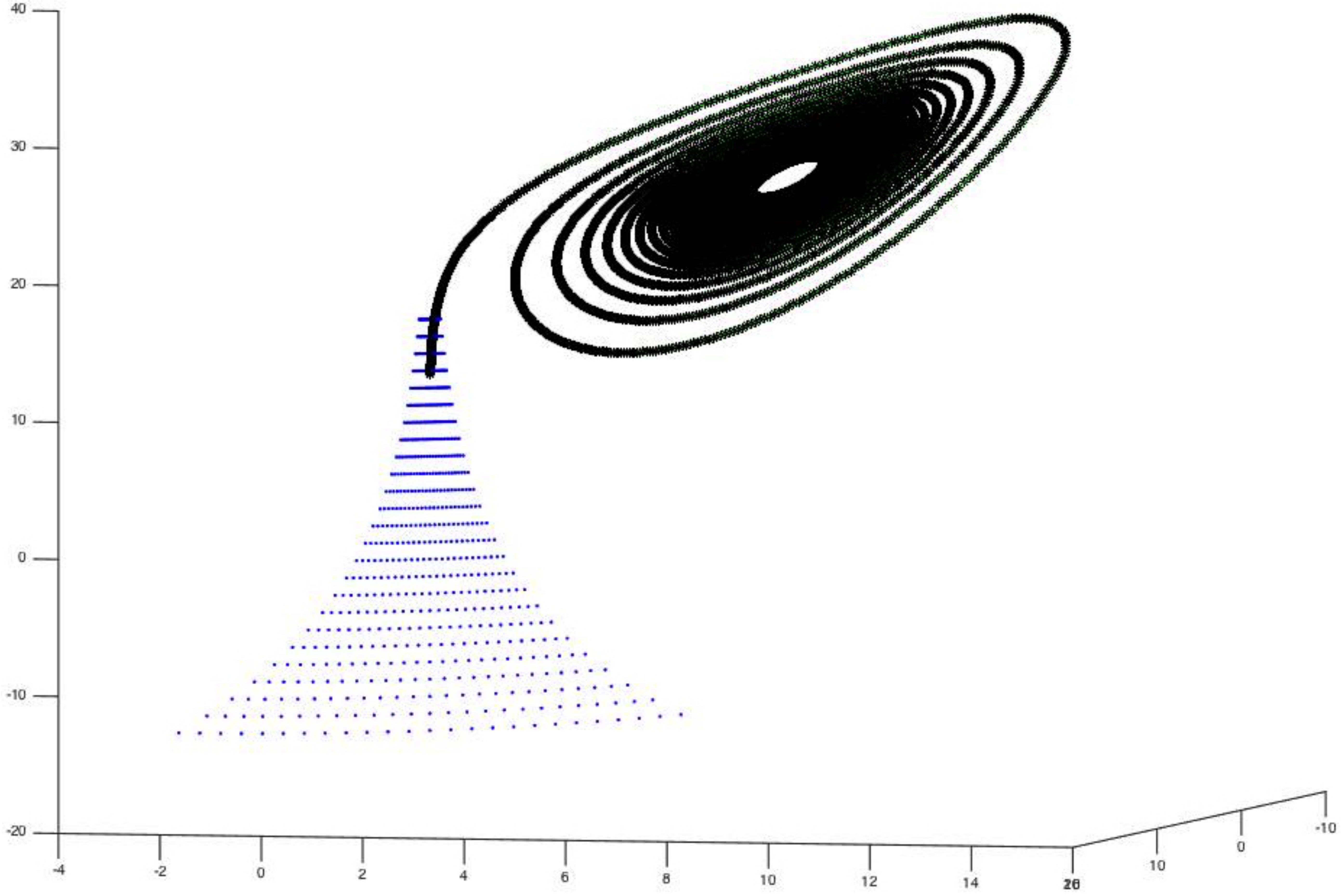
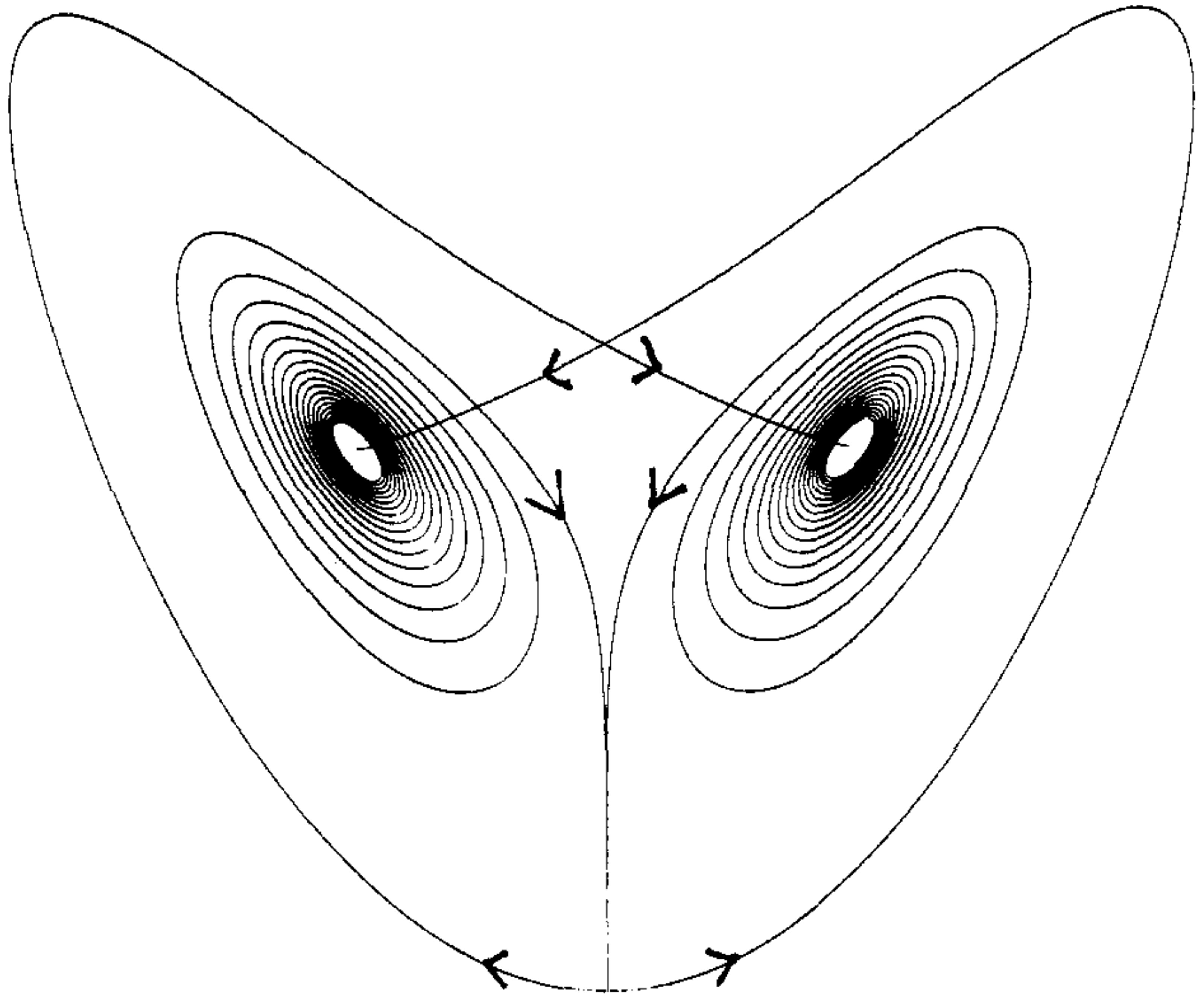
$$F(\lambda, \mu, T, \sigma, \rho, \mathbf{v}_0, P, Q, R) = \begin{pmatrix} P(r) - Q(-\tau) \\ Q(\tau) - R(-r) \\ \|P'(0)\|^2 - C \\ \|R'(0)\|^2 - C \\ \lambda u P'(u) - f_{\sigma,\rho}(P(u)) \\ Q(t) - \mathbf{v}_0 - T\int_0^t f_{\sigma,\rho}(Q(s))\, ds \\ \mu s R'(s) - f_{\sigma,\rho}(R(s)) \end{pmatrix}$$

The set up is inspired by a similar approach developed in

MR4068579 **Reviewed**
van den Berg, Jan Bouwe (NL-VUAM-M); Sheombarsing, Ray (NL-VUAM-M)
Validated computations for connecting orbits in polynomial vector fields. (English summary)
Indag. Math. (N.S.) 31 (2020), no. 2, 310–373.
37C29 (34C37 35C07 35K40 35K57 37M99)

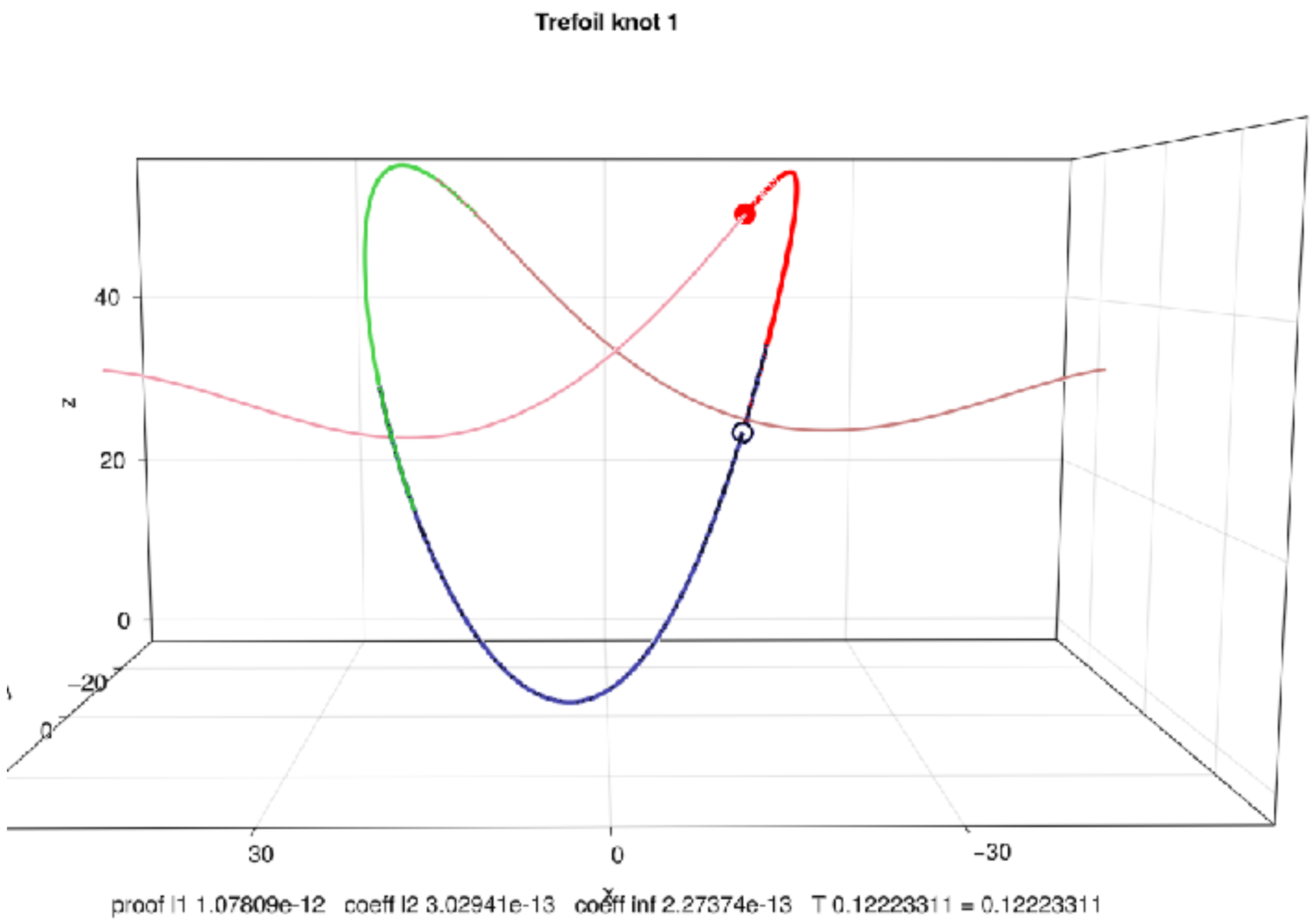
T-points: The transverse connections can be proven using the techniques developed in

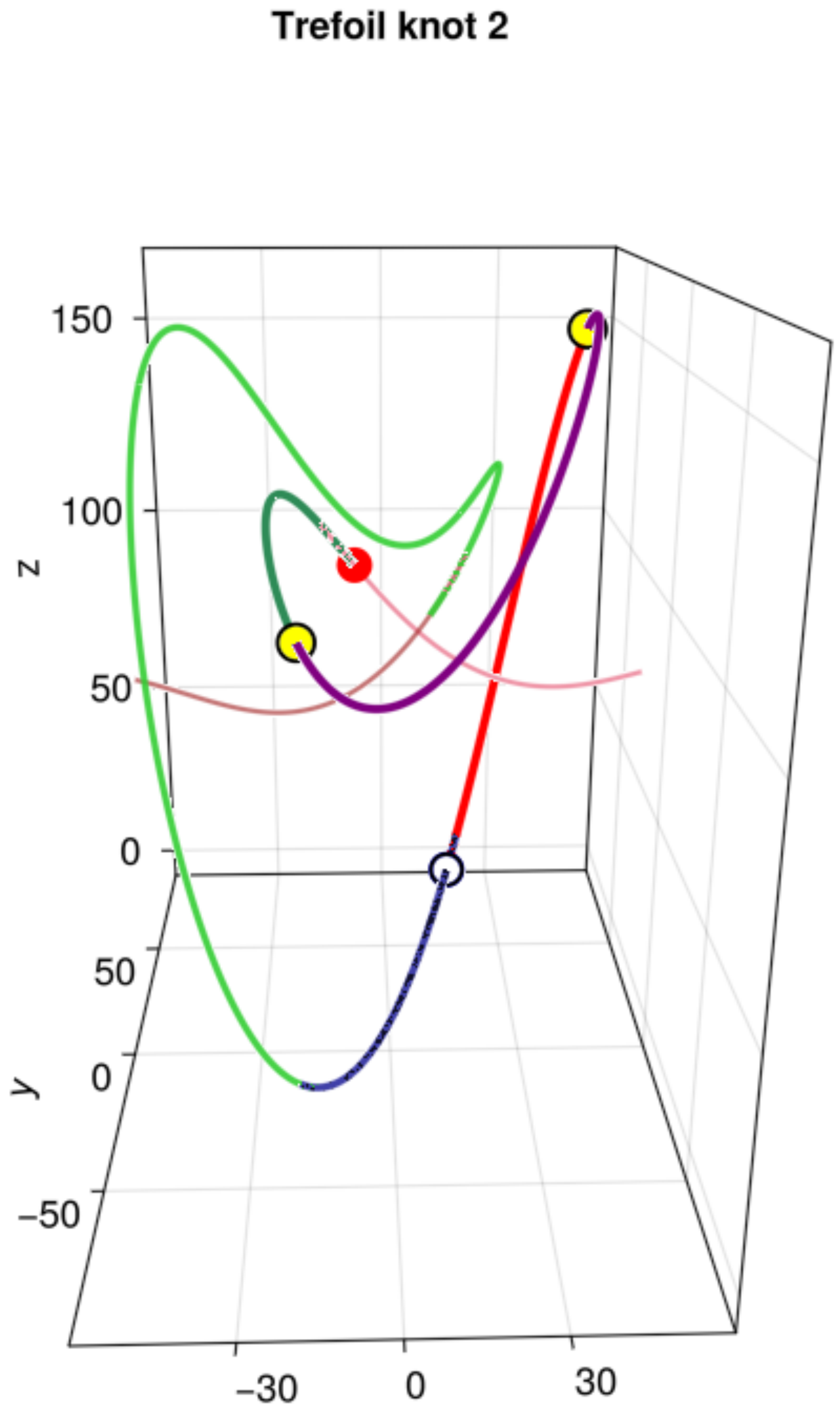




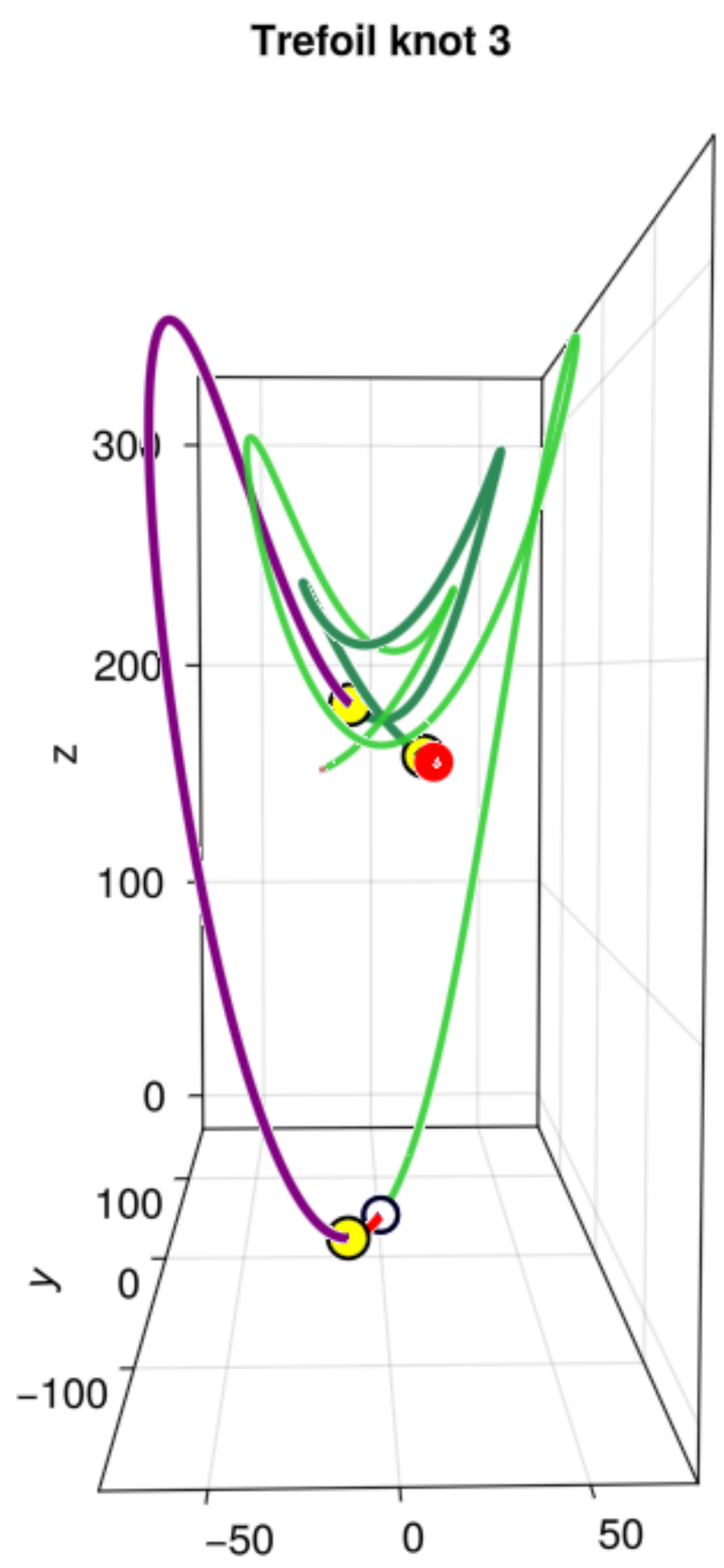
MR3792792 JDMJ Validated numerics for equilibria of analytic vector fields: invariant manifolds and connecting orbits. *Rigorous numerics in dynamics,* 27–80,
Proc. Sympos. Appl. Math., 74, *Amer. Math. Soc., Providence, RI,* 2018.

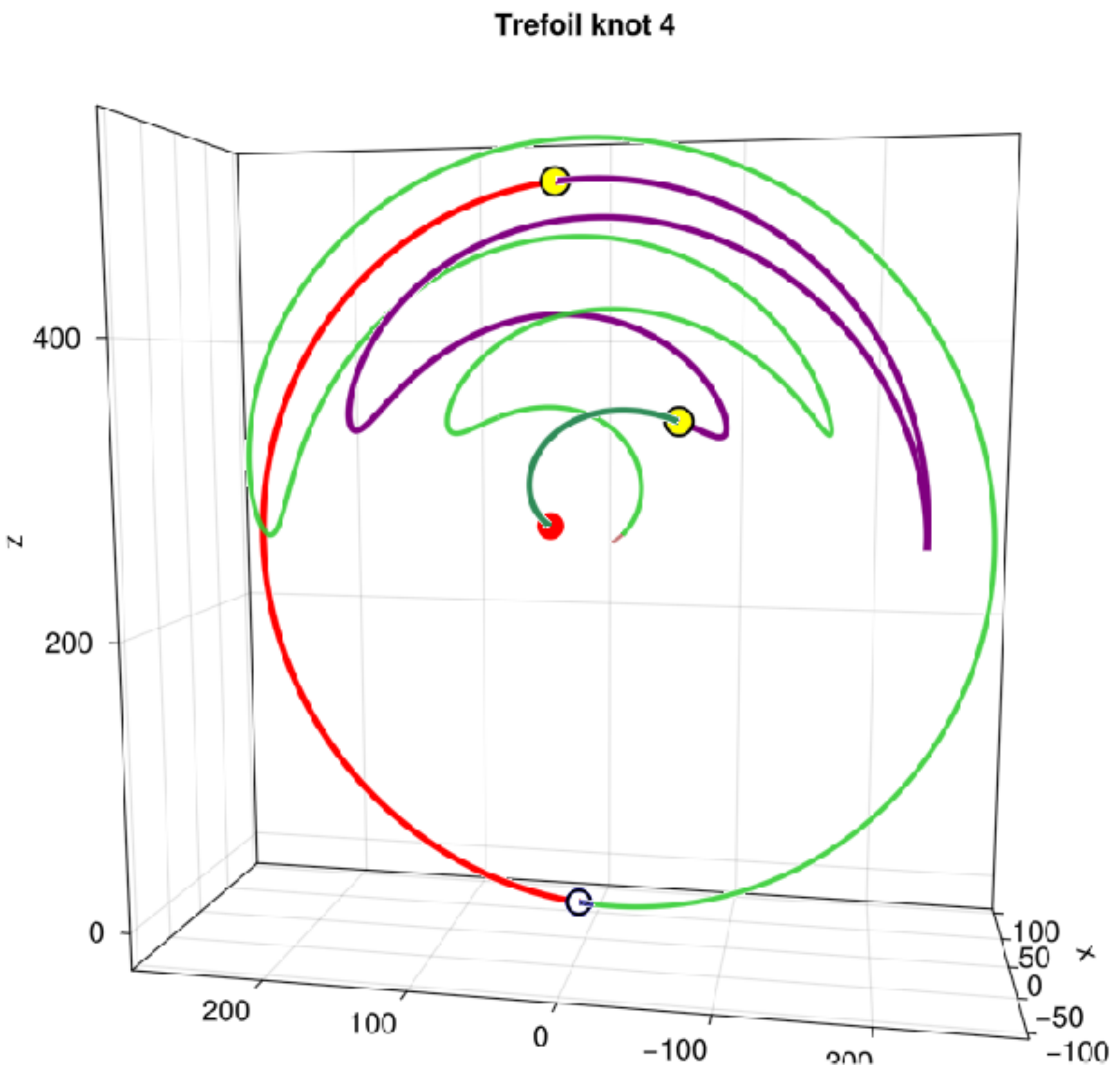
T-points: The transverse connections can be proven using the techniques developed in

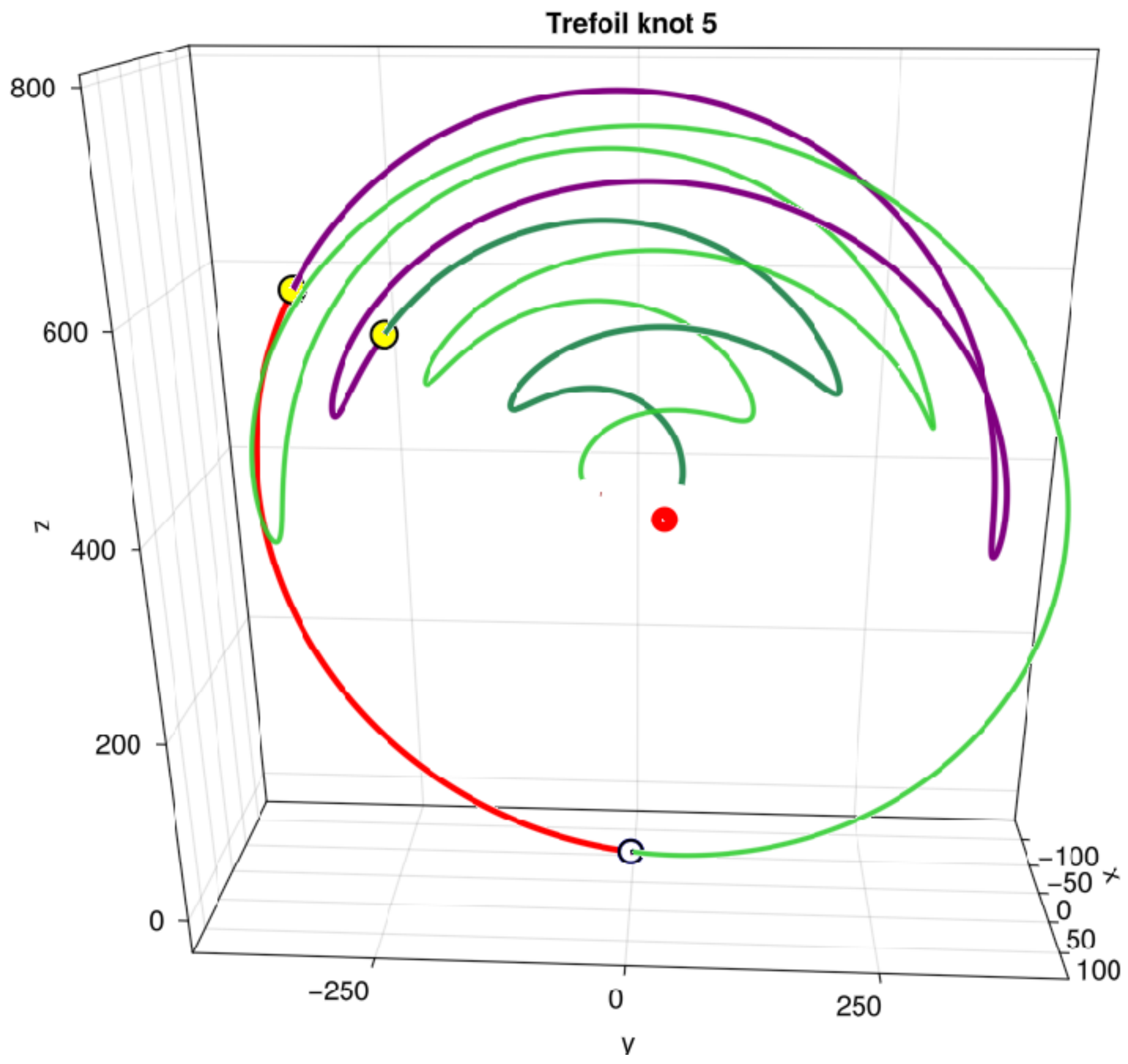




T-points: The transverse connections can be proven using the techniques developed in





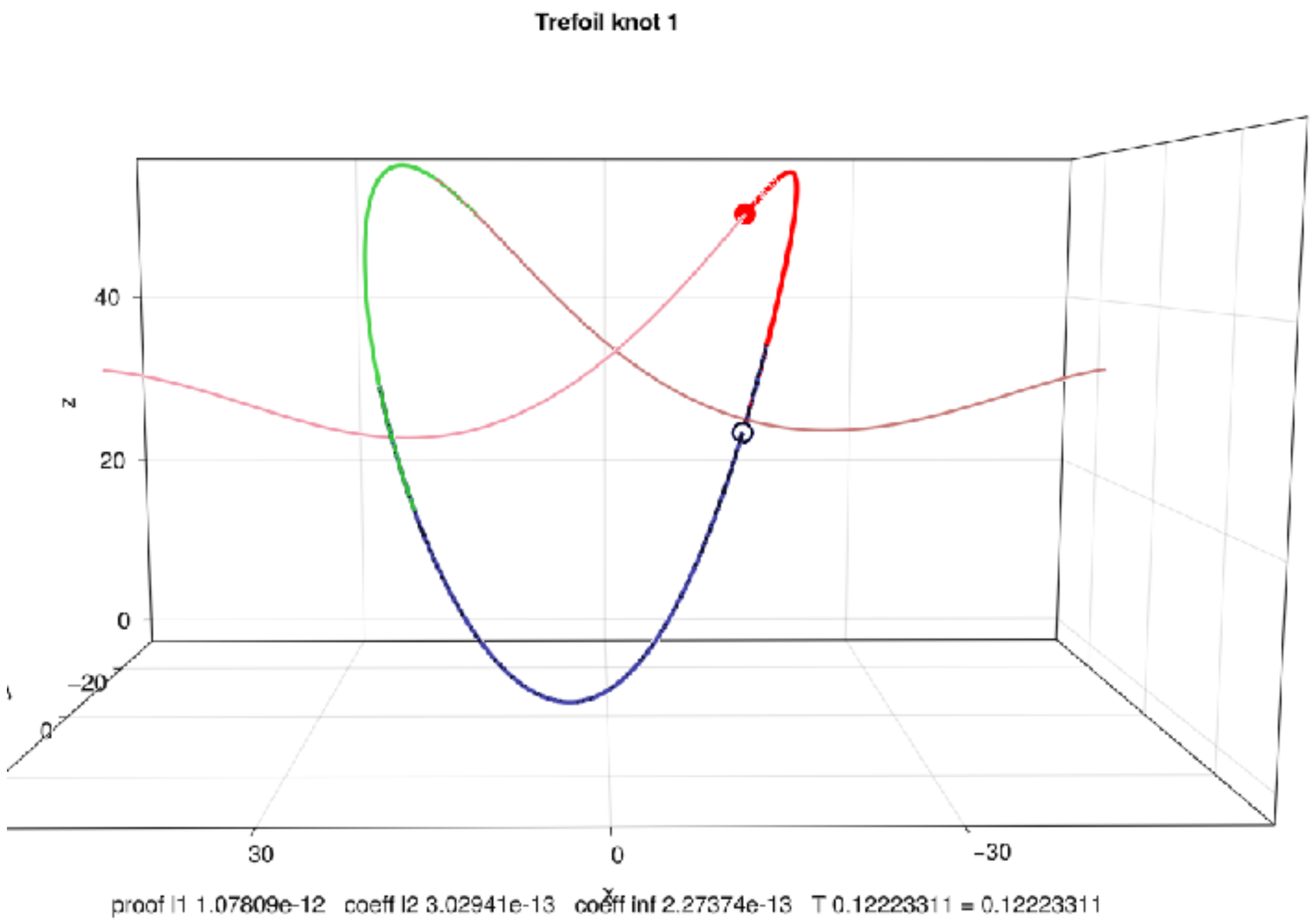


Future work and extensions:

Prove it's a trefoil.

Other systems/bifurcations?

Try to understand the boundary between CAP values and perturbative results.



Thank you to the organizers!

Thank you all for listening!

A specific example problem: T-points in the Lorenz system.

T-points: The transverse connections can be proven using the techniques developed in

MR1961956 Wilczak, Daniel; Zgliczynski, Piotr
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Rigorous numerics for symmetric connecting orbits: even homoclinics of the Gray-Scott equation.
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A computer-assisted proof of the existence of solutions to a boundary value problem
with an integral boundary condition. *Commun. Nonlinear Sci. Numer. Simul.* 16 (2011), no. 3, 1227–1243.

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MR3579307 Barker, Blake; Zumbrun, Kevin
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Computer-assisted proof of Shil'nikov homoclinics: with application to the
Lorenz-84 model. *SIAM J. Appl. Dyn. Syst.* 16 (2017), no. 3, 1453–1473.

MR3871847 Capiński, Maciej J.; Turaev, Dmitry; Zgliczyński, Piotr
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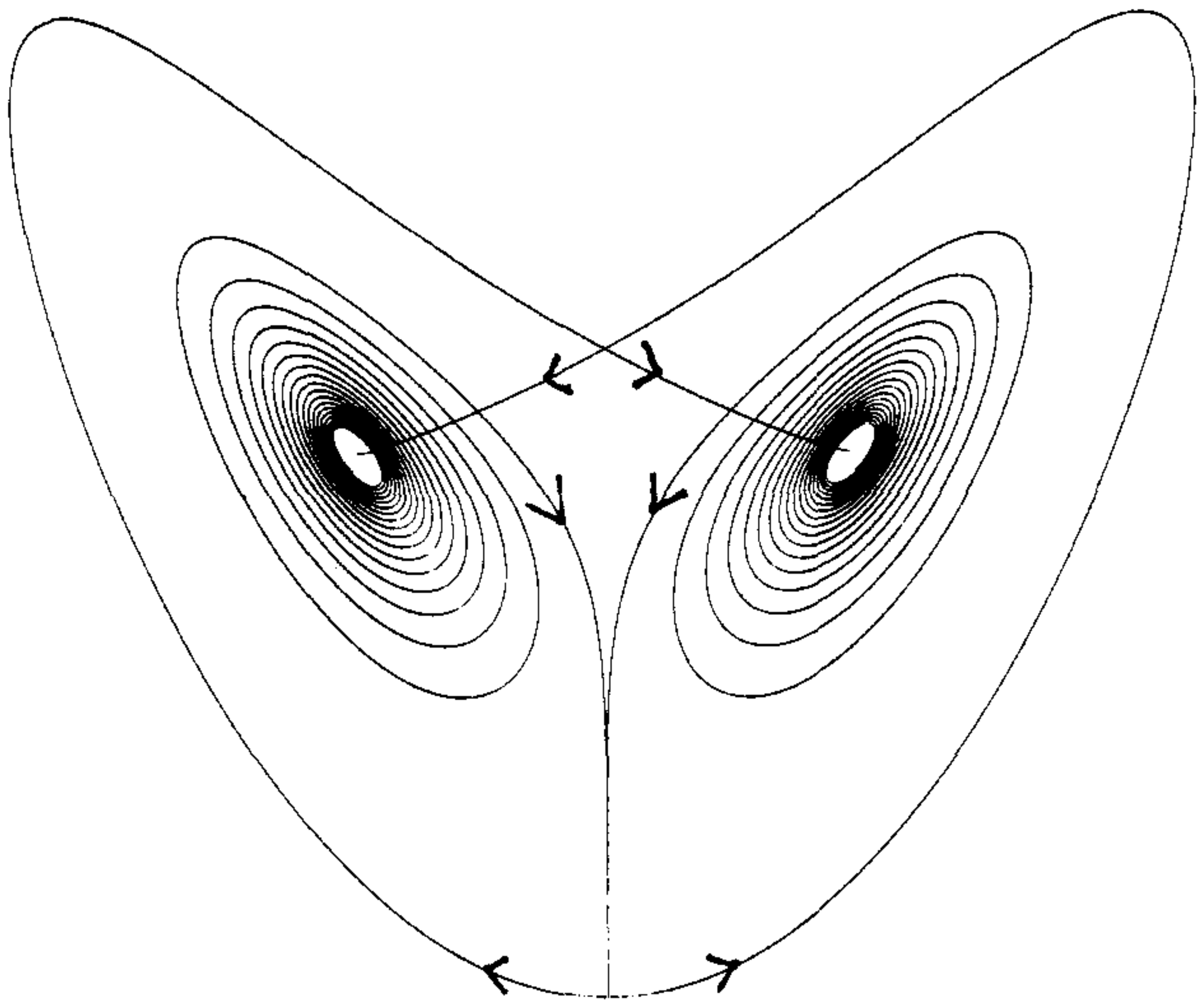
MR3919451 Kepley, Shane; JDMJ
Homoclinic dynamics in a restricted four-body problem: transverse connections for the saddle-focus equilibrium solution set.
Celestial Mech. Dynam. Astronom. 131 (2019), no. 3,

MR4417102 van den Berg, Jan Bouwe; Duchesne, Gabriel William; Lessard, Jean-Philippe
Rotation invariant patterns for a nonlinear Laplace-Beltrami equation: a Taylor-Chebyshev series approach.
J. Comput. Dyn. 9 (2022)



T-points:

Barrio R, Shilnikov A L and Shilnikov L 2012 Kneadings, symbolic dynamics and painting Lorenz chaos *Int. J. Bifurcation Chaos* **22** 1230016

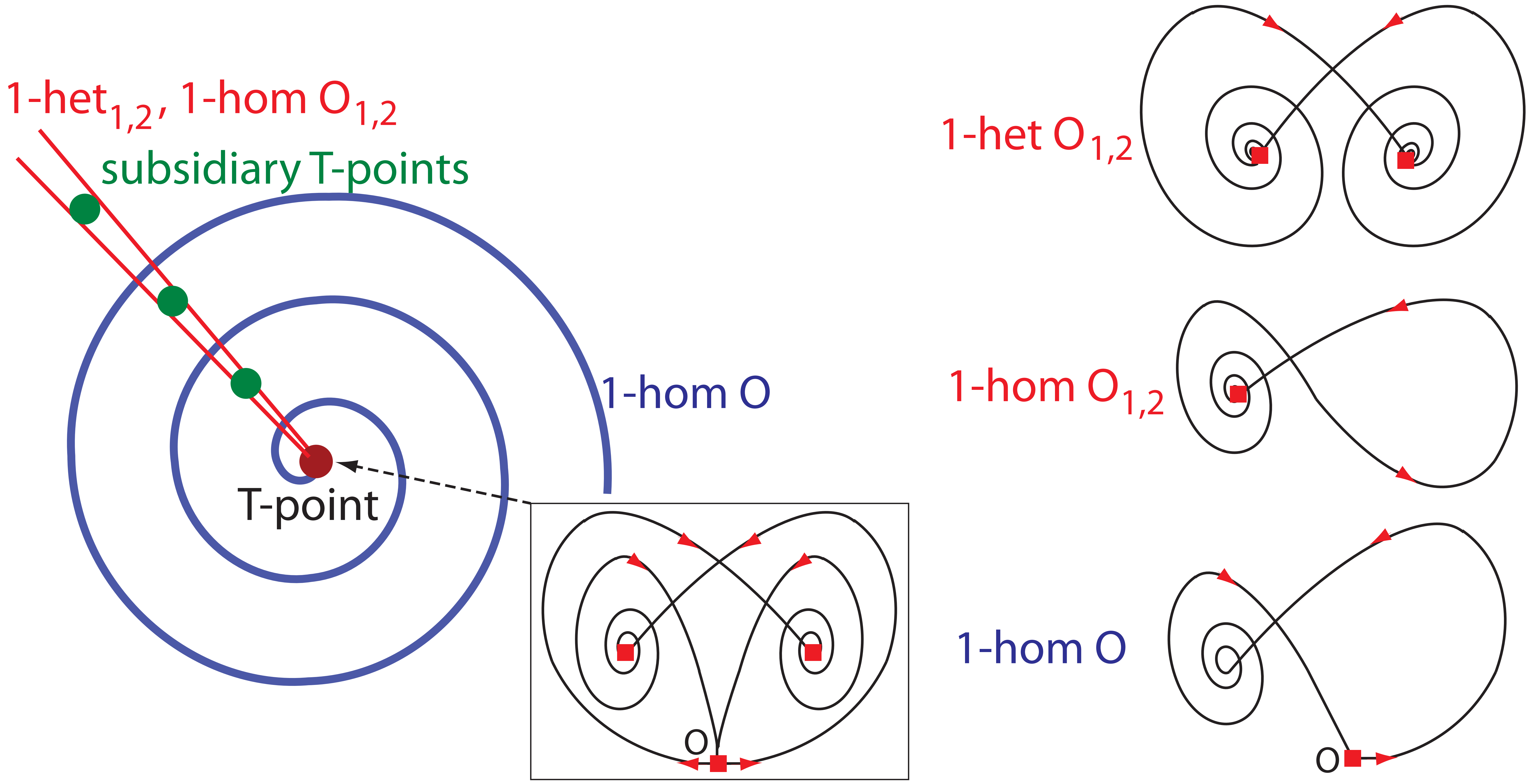


Figure 1. Caricature of the bifurcation unfolding of the ordinary T-point for symmetric Lorenz-like systems with a closed heteroclinic connection evolving both saddle-foci and the saddle. Point out that with each revolution approaching the T-point along the curve (1-hom O), the number of turns of the one-dimensional separatrix of the saddle, O, around the saddle-focus increases by one in the homoclinic loop and becomes infinite at the T-point.